**Mechanics**

**Distance, velocity, acceleration and simple calculus.**

1 The total length of traffic passing per lane in 1 hour is 96000 m, and this contains 2400 vehicles, accounting for 11040 m, and leaving 84960 m unoccupied by cars, i.e. 35.4 m per car. (Note the gap recommended by the Highway Code is about 50 m at this speed).

2 If acceleration is constant we have $v(t) = u + \int_0^t a dt = u + at$ and

$s(t) = \int_0^t v(t')dt' = \int_0^t u + at'dt' = ut + \frac{1}{2}at^2$.

$v^2 = (u+at)^2 = u^2 + 2uat + a^2t^2 = u^2 + a\left(2ut + at^2\right) = u^2 + 2as$ as required.

In terms of energy, final KE = initial KE + work done, i.e. $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + Fs$ (*F* is a constant because the acceleration is constant), and since $F = ma$, this becomes $\frac{1}{2}mv^2 = \frac{1}{2}mu^2 + mas$, cancelling *m* and doubling gives the required expression.

3 If the car brakes, it will come to a halt after travelling a distance $-u^2/2a$, which is 47.6 m, i.e. short of the light, and this will take time $-u/a$, i.e. 4.8 s, so the car will stop before the light, after it has changed to red.

If the car accelerates, then when the light changes it will be travelling at $v = u + at = 23.8$ m s$^{-1}$ and will have travelled a distance $s = \left(v^2 - u^2\right)/2a = 54.7$ m, i.e. it will have passed the light when it changes but only be 4.7 m into the junction. The driver should brake.

4 The easy way to do this is to calculate how the time to the crash and work out how far the fly flies in this time (much easier than the infinite series you would otherwise need). The fly flies twice as fast as each train and so it will cover double the distance of a train. At the crash, each train has covered 500 m, which takes 50 s at the given speeds. The fly has therefore flown 1 km.

5 (a) $v_x(t) = \frac{dr_x}{dt} = 3\alpha t^2$ and hence $a_x(t) = \frac{dv_x}{dt} = 6\alpha t$

(b) $v_x(t) = v_x(0) + \int_0^t a_x(t')dt' = 3\alpha t^2$ and integrating again $r_x(t) - r_x(0) = \int_0^t v_x(t')dt' = \alpha t^3$.

**Vectors (position, velocity, acceleration).**

6 Assuming the Professor to have a height of 1.8 m, the egg must drop 44.2 m.
The egg starts at rest and accelerates with a constant acceleration of $g = 9.8 \text{ m s}^{-2}$,
at the moment of impact it is therefore travelling at $v = \sqrt{2gs} = 29.4 \text{ m s}^{-1}$.
The time taken to accelerate to this speed is $t = v/g = 3.00$ s.
In 3.00 s the professor covers 3.60 m.

7 The winger will be at the vector sum of all the displacements and his initial position,
i.e. (9,−5) + (0,15) + (−6,4) + (12,20) = (15,34).
The pass must therefore follow the vector (15,34) – (0,–8) = (15,42), which has length 44.6 m.

8 The distance between every pair of points is the same, i.e. $\sqrt{2}$. The only body with 4 corners where all 6 distances are equidistant is a regular tetrahedron. Each triangular face must be equilateral.
Taking the scalar product $\vec{\mathbf{a}}.\vec{\mathbf{b}} = ab\cos\theta$ $(1,1,1).(1,-1,-1) = -1 = \sqrt{3}\sqrt{3}\cos\theta$. Hence $\theta =$ 1.91 radians or 109.5 degrees.

9 The velocity relative to the air is $\mathbf{v}_{plane} - \mathbf{v}_{air}$, and this is $(-297,297)$ knots (420 knots SE).
Hence $\mathbf{v}_{plane} = \mathbf{v}_{plane} - \mathbf{v}_{air} + \mathbf{v}_{air} = (-297,297) + (0,120) = (-297,417)$ and the ground velocity is 512 knots with a bearing of 125° (measured clockwise from North, for air navigation).

10 (a) The rmms are 141.94 and 39.09. $\mathbf{v}_{cm} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = (313.6, 64.8) \text{ m s}^{-1}$

(b) $\mathbf{v}_{Mel} - \mathbf{v}_{cm} = (86.4, -64.8)$ m s$^{-1}$

$\mathbf{v}_{K} - \mathbf{v}_{cm} = (-313.6, 235.2)$ m s$^{-1}$

(c) $\mathbf{p}_{Mel} - \mathbf{p}_{cm} = (2.04, -1.53) \times 10^{-23}$ kg m s$^{-1}$

$\mathbf{p}_{K} - \mathbf{p}_{cm} = (-2.04, 1.53) \times 10^{-23}$ kg m s$^{-1}$

They are equal and opposite because from the definition of the centre of mass $(m_1 + m_2)\mathbf{v}_{cm} = m_1\mathbf{v}_1 + m_2\mathbf{v}_2$, i.e. $m_1(\mathbf{v}_1 - \mathbf{v}_{cm}) + m_2(\mathbf{v}_2 - \mathbf{v}_{cm}) = 0$.

11 (a) The horizontal speed is not altered, thus the time taken to travel 2 horizontal cm is $s/u = 8.33$ ns. During this period they experience a constant upward acceleration and therefore cover a vertical distance $\frac{1}{2}at^2$ = 1.39 cm.

(b) The velocity is $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2.40, 3.33) \times 10^6$ m s$^{-1}$, which is directed 54.2° upwards from horizontal.

12 (a) Equations that describe how the system evolves through time.

(b) i. $\frac{dv_x}{dt} = 0$, $\frac{dx}{dt} = v_x$, $\frac{dv_z}{dt} = -\alpha$, $\frac{dy}{dt} = v_y$

ii. The solutions to the equations of motion are

$v_x = v_0 \cos\theta$, and $x = v_0 t \cos\theta$

$v_z = v_0 \sin\theta - \alpha t$ and $z = v_0 t \sin\theta - \frac{1}{2}\alpha t^2$

The time spent in the region is therefore $t = \frac{L}{v_0 \cos\theta}$, during which the *z* displacement is

$z = v_0 \frac{L}{v_0 \cos\theta} \sin\theta - \frac{1}{2}\alpha \left( \frac{L}{v_0 \cos\theta} \right)^2$. For this to be zero we must have $v_0 \sin\theta = \frac{1}{2}\alpha \frac{L}{v_0 \cos\theta}$,

or equivalently $\alpha = \frac{2v_0^2}{L} \sin\theta \cos\theta$, as required.

iii. $v_x$ is unchanged, and $v_z = v_0 \sin\theta - \alpha \frac{L}{v_0 \cos\theta}$, but since $\alpha = \frac{2v_0^2}{L} \sin\theta \cos\theta$, this becomes $v_z = -v_0 \sin\theta$, as required.